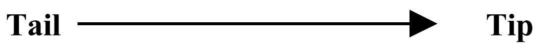
Relative Motion

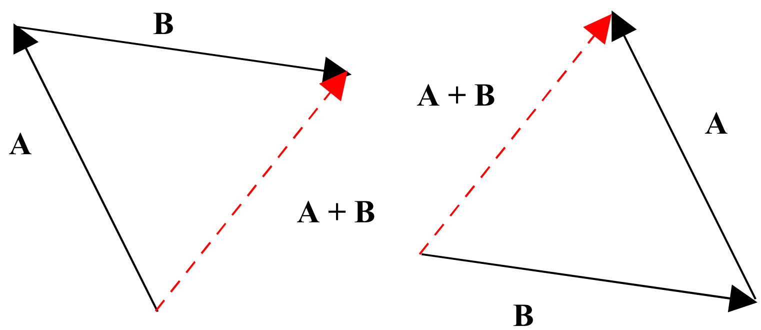
Relative motion is a difficult concept most students have a hard time visualizing.
Some basic vector rules must need to be learned first to help understand the concept.

A Vector has a magnitude(length) and a direction. We say that a vector has a tip and a tail. They are defined as below.



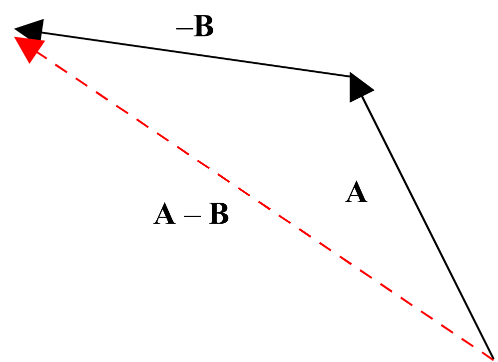
Adding Vectors

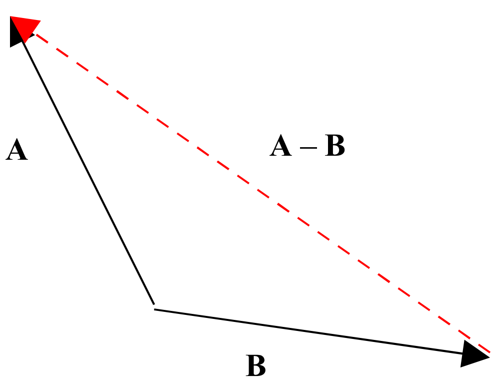
Too add Vectors you always add the vectors "tip to tail". The resultant starts from the tail of the first to the tip of the second.
Note: that the A + B = B + A

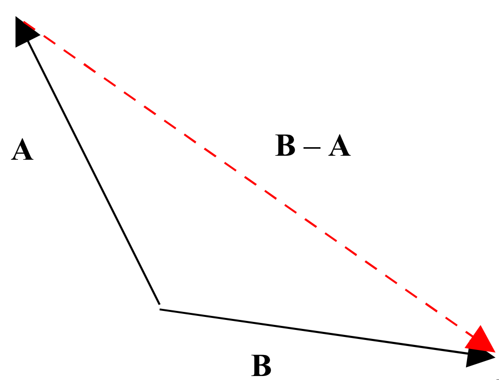


Subtracting Vectors

Too subtract vectors you always often see the subtraction turned into an addition as follows. **A – B** = **A + (– B)** . The resultant is as follows. However I find when working with two vectors I find the "tail to tail" method quicker. When using the tail to tail method the resultant is drawn from the second vector to the first. This method is ideal is showing that **A – B** and **B – A** is the same Magnitude but in the opposite direction since we simply draw the vector in the opposite direction.







Notation

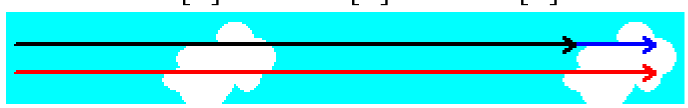
To help solve the relative motion questions we need to understand a notation that can be used to explain the motion.
First the vector equation AB + BC + CD = AD can be understood as meaning: Walk from point A to point B then from point B to point C then point C to point D can be simplifies to walking from point A to point D. A rule to follow is if we are adding and they adjacent letters are the same we can "join" the vector together. i.e. AB + BC + CD = AC + CD = AD
Having introduced this concept we can now get into understanding relative motion.

THE VECTORS
When analyzing relative motion we realize there are 3 vectors involved.
They are as follows.
${}_{a}\mathbf{V}_{g}$ = Velocity of the air with respect to the ground. This represents if you are standing still on the ground what velocity does the wind seem to have.
${}_{p}\mathbf{V}_{g}$ = Velocity of the plane with respect to the ground. This represents what velocity does the plane (think of it as following its shadow) on the ground.
${}_{p}\mathbf{V}_{a}$ = Velocity of the plane relative to the air. This is a harder one to visualize. But think of it as if you stick your hand out the window of a car. You sense you have a velocity relative to the air. If you did that while traveling in a hot air balloon you wouldn't feel anything since you are traveling only as fast as the wind? So your relative velocity is zero.

How are these three vectors related?
Using the relationship introduced earlier we see that
$${}_{p}\mathbf{V}_{a} + {}_{a}\mathbf{V}_{g} = {}_{p}\mathbf{V}_{g}$$

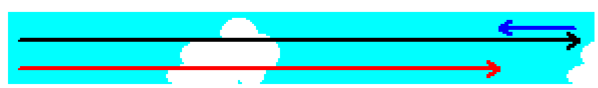
This equation makes sense. The velocity of a plane respect to the ground is the velocity of the plane relative to the air plus the velocity of the air relative to the ground.

Example. If the wind has a velocity of 10 m/s [E] and the plane flies at a velocity of 60 m/s [E] relative to the air, the velocity relative to the ground is 10 m/s [E] + 60 m/s [E] = 70 m/s [E]. The speed is faster since than the plane can fly since the wind helps "speed up" of the plane.



The important thing to realize is we will always add when we are finding the ground velocity. This is true even if the wind is in the opposite direction. The resultant vector is offset to show how you line up the vectors to calculate them. Remember. **Add:** Tip to Tail. **Subtract**: Tail to Tail.

Example: If the wind has a velocity of 10 m/s [W] and the plane flies at a velocity of 60 m/s [E] relative to the air, the velocity relative to the ground is 10 m/s [W] + 60 m/s [E] = –10 m/s [E] +70 m/s [E] = 60 m/s [E].



Notice how we still put the vectors tip to tail.

If we do know the ground velocity, by looking at the equation and rearrange it we realize that we will be subtracting to find the missing value. So we will be putting the vectors tail to tail.

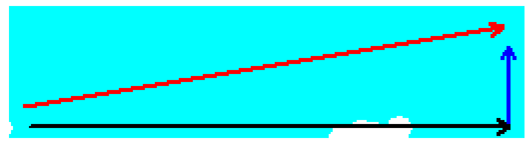
Example: If the wind has a velocity of 10 m/s [E] and the plane has a velocity of 60 m/s [E] relative to the ground, the velocity relative to the air is 60 m/s [E] - 10 m/s [E] = 50 m/s [E]. Notice how vectors are tail to tail and resultant is 2nd vector to first.

Likewise

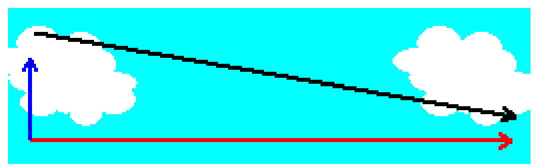
Example: If the wind has a velocity of 10 m/s [W] and the plane has a velocity of 60 m/s [E] relative to the ground, the velocity relative to the air is
60 m/s [E] - 10 m/s [W] = 70 m/s [E].

The understanding of these rules are more important as we go into two dimensions.
Lets repeat the examples but set the wind going North. Rules still follow but the final answer requires a little more work in calculating. Remember magnitude and direction is needed.

Example. If the wind has a velocity of 10 m/s [N] and the plane flies at a velocity of 60 m/s [E] relative to the air, the velocity relative to the ground is
10 m/s [N] + 60 m/s [E] = 60.8 m/s[E 9.5^{o} N].



Example: If the wind has a velocity of 10 m/s [N] and the plane has a velocity of 60 m/s [E] relative to the ground, the velocity relative to the air is 60 m/s [E] - 10 m/s [N] = 60.8 m/s [S 80.5 E].



Use the programs built in Questions to practice using vectors of different directions. As well, do one of the Relative motion work sheets